or an ion drag ratio of $(1 - e\varphi_0/E)$ We can see from the curves that the empirical equation fulfills these limiting conditions

Conclusions

In conclusion it can be said that 1) the electric drag can be a significant part of satellite ion drag; 2) a theory for electric drag must include both impacting of ions on the surface and the shielding effect of the plasma sheath; 3) the drag produced by nonimpacting ions is important for values of a/λ_D below about 10 but can be neglected for most practical satellite conditions; and 4) a simple empirical relation is available which permits rapid estimation of electric drag of spheres over practical ranges of interest It is hoped that these results may resolve some major questions regarding electric drag and reduce the problem to questions of the proper values of satellite potential and of fraction ionization of the atmosphere

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Oxygen-Electron Attachment in Hypersonic Wakes

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THE significance of electron attachment by molecular oxygen on the decay of electron densities in hypersonic wakes has been demonstrated both experimentally[1] and theoretically [2–6] The purpose of the present note is to show the relation between the electron density decay resulting from the attachment effect and the wake velocity decay

The attachment reaction is†

$$O_2 + O_2^- \underset{K_R}{\overset{K_F}{\rightleftarrows}} 2O_2 + e^- \tag{1}$$

in which [7 8]

$$K_F \simeq 2(10)^{10}T^{3/2}e^{-5337/T} \text{ cm}^3\text{/mole-sec} \tag{2a}$$

$$K_R \simeq 10^{18} \text{ (cm}^3)^2\text{/mole}^2\text{-sec} \tag{2b}$$

where T is in degrees Kelvin

The attachment process has been found to be effective once the wake temperature drops to 500°–800°K for altitudes of 150 kft and below [2–6] Curves A and B of Fig 1 show such effects on the axis electron density of the downstream turbulent wake of a 1-ft base radius, 10° cone flying at 23 kft/sec at an altitude of 120 kft [5 10] The electron removal mechanism for curve A is recombination, i e ,

$$N + O \rightleftarrows NO^+ + e^- \tag{3}$$

whereas for curve B both recombination and attachment are

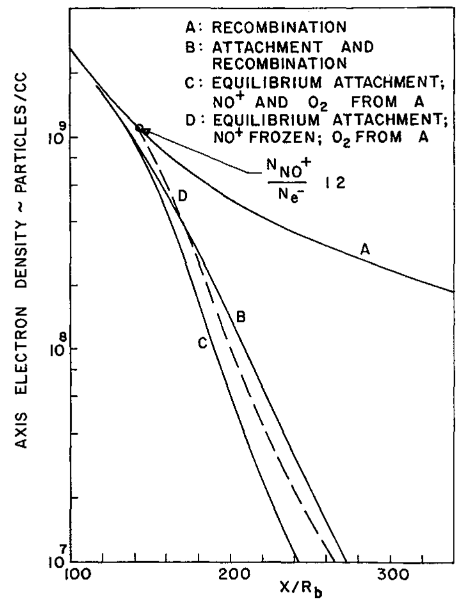

Fig 1 Effect of attachment on axis electron density; 10° cone, R_b = 1 ft, 120 kft, turbulent wake

active; it is clear that attachment effects are dominant in the downstream regions The dominating effect of attachment has been observed by Labitt in his ballistic range experiments[1] and has been attributed to reaction (1) going into a "near-equilibrium" condition [1 3 10] This is also indicated here by curve C of Fig 1 The curve was calculated by allowing reaction (1) to be in equilibrium; the O_2 and NO^+ concentrations were obtained from the curve A calculations It is evident that C is a reasonably good approximation to B in terms of locating the streamwise position of a given electron density level Also, it is anticipated that at lower altitudes the approximation improves

The foregoing suggests that, once reaction (1) is initiated, the recombination may be regarded as being chemically frozen relative to the attachment Curve D of Fig 1 demonstrates this situation; there the attachment reaction was put into equilibrium while the NO^+ concentration was determined by pure diffusion starting at the point where the attachment begins to dominate The latter was arbitrarily selected to be the location where the curve B calculations show 20% more NO^+ particles than electrons The D calculations used the O_2 concentrations of A; however, since both A and B have almost identical O_2 populations, this assumption is valid It is seen that D predicts the location of a specified electron density to within a few percent Similar results are obtained when the definition of the location of the point at which attachment begins to dominate is modified [10]

The conclusions just reached may be used to develop a scaling law for the electron density in the attachment dominated regions of wakes First, with reaction (1) in equilibrium, its mass-action law combined with charge balance between O_2^-, e^-, and NO^+ yields

$$N_{e^-} \sim \frac{\alpha_{NO^+}}{1 + C(K_R/K_F)\rho\alpha O_2} \tag{4}$$

where α_i represents the mass fraction of species i, ρ is the mass density, and C is a constant

Recognizing that the attachment process becomes effective in the low velocity and temperature region of the wake where the mass fractions of the thermodynamic state determining species and the stagnation enthalpy are essentially constant, the following simplifications apply:

$$T \simeq T_\infty + (U_\infty U/C_P) \tag{5}$$

$$\rho \sim P_\infty/T \tag{6}$$

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† Charge exchange between NO^+ and O_2^- is also required [9]

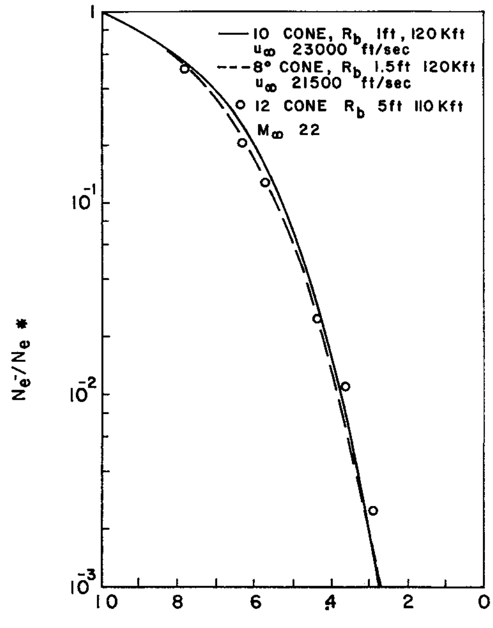

Fig 2 Correlation of axis electron density with axis velocity (turbulent wakes)

where P_∞ is the pressure, U_∞ is the flight velocity, and U is wake velocity in earth-fixed coordinates (i e , the velocity defect) As just noted, the NO^+ population may be considered to be controlled by diffusion alone; hence,

$$\alpha_{NO^+} \sim U \tag{7}$$

Combining (4–7), there results

$$N_{e^-} \sim \frac{U}{1 + C_1 P_\infty (1 + C_2 U)^{-5/2} \exp[C_3/(1 + C_2 U)]} \tag{8}$$

where the various constants and the T_∞ and U_∞ dependent terms have been incorporated into C_1, C_2, and C_3 Therefore, neglecting differences in T_∞ and U_∞, the electron density decay is governed by the velocity decay at a given altitude, i e , at fixed P_∞,

$$N_{e^-}/(N_{e^-})^* = F(U/U^*) \tag{9}$$

where $(N_{e^-})^*$ and U^* are the conditions at the initiation of the attachment effects

Figure 2 demonstrates the application of Eq (9) to the axis properties of the wakes of a 10° cone (R_b = 1 ft, U_∞ = 23,000 fps) at 120 kft,[10] an 8° cone (R_b = 1 5 ft, 21,500 fps) at 120 kft,[11] and a 12° cone (R_b = 0 5, M_∞ = 22) at 110 kft [3] The $(N_{e^-})^*$ and U^* were taken to be their respective values at the location where the attachment effects have had a 20% effect on the electron density (i e , the same definition as just used) The data taken from Ref 3 apply to a "model gas" based on oxygen properties; however, air characteristics are accurately reproduced Considering the differences in flight velocity for the three cases, and that one case applies to a slightly different altitude, the correlation of the data is good

Although the scaling law is somewhat limited in that it requires the upstream flow properties (N_{e^-*} and U^*) as inputs,‡ it does help to clarify the details of the final decay of electron population Furthermore, since the velocity governs the process, the present results emphasize the importance of an accurate representation of the fluidmechanical properties of the downstream wake

‡ However, binary scaling is expected to correlate the upstream (recombination-controlled) region

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Tetrahedron Elements in the Matrix Force Method of Structural Analysis

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Introduction

THE use of solid tetrahedron elements in matrix structural analysis is very atttractive, since such elements can be employed in the idealization of solid structural components The tetrahedron elements so far have been used only in the displacement method of analysis,[1, 2] in which the element forces were related to the corresponding displacements through the stiffness matrices determined on the assumption of linearly varying displacements within the tetrahedron This assumption leads to a compatible, constant stress distribution that also satisfies the stress equilibrium equations within the tetrahedron Since the stresses vary from element to element, the boundary stress equilibrium will, in general, be violated The over-all equilibrium of all the elements, however, is maintained through the equilibrium of element forces at common joints, i e , vertices of the tetrahedra

The assumption of constant stresses within the element can also be used on the solid tetrahedron to establish its flexibility properties required in the matrix force method of analysis The flexibility properties of the tetrahedron element may be determined most conveniently for a set of edge force systems acting along the six edges of the tetrahedron The concept of edge forces was used in Ref 3 to determine flexibility properties of the triangular plate element, where the edge force systems were acting along the three sides of

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